INSTRUCTIONS: 100 questions, 10 minutes. Do not open packet until instructed to do so. The problems have been split into 6 sections: 25 True/False, 20 Arithmetic, 15 Counting/Probability, 15 Geometry, 15 Algebra, and 10 Miscellaneous. No justification for any problems is required. Correct answers will be awarded 1 point regardless of difficulty and wrong or blank answers will receive 0 points. Feel free to skip to any section. Problems within each section are ordered in roughly increasing difficulty (please do move on if you get stuck).
Please write the entire word true or false for your answers on the true/false section.
Opportunity for extra points on true/false: If you get every single question on the T/F section wrong (with no blanks), you will get 50 points for that section (only valid for true/false).

1 True/False

1. This test has 100 questions!
2. $3^4 > 4^3$.
3. $5 > 10$ OR $5 < 7$.
4. For all real numbers a, b, c, the equivalence $a^{b^c} = (a^b)^c$ holds.
5. Flipping two heads out of two fair coin tosses is more likely than rolling a 5 or 6 on a 6-sided die.
6. A sphere with diameter 1 has numerically greater volume than the area of a circle with diameter 1.
7. Suppose Alex and Sam ran in a 2-mile race. Alex ran at a pace of 5 miles/hour for the first mile, then ran for 9 miles/hour for the second. Sam on the other hand, ran 7 miles/hour for both miles. Alex crossed the finish line first.
8. It is possible to put 6 X's on a tic tac toe board without making three in a row in any direction.
9. There is precisely one three-digit number with decimal representation ABC such that fourteen times BC is one less than ABC.
10. The digit K (in base 100) is also known as 21 in base 10.
11. A triangle with sides of length 5, 5, and 8 has a greater area than a triangle with sides of length 5, 5, and 6.
12. There are infinitely many pairs of integers (m, n) satisfying $m^2 + n^2 + 1 = 16^{m+n}$.
13. p and q are positive integers such that $\frac{p}{q} = \frac{2}{3}$. The expression $\frac{p+2}{q+3}$ must always have value $\frac{2}{3}$.
14. The longest arithmetic sequence of primes has length 5 (for example, 5, 11, 17, 23, 29).
15. The minimal number of cuts required to divide a cake into 8 pieces is 4.
16. There are less than three distinct (positive) perfect cubes in the Fibonacci sequence.
17. There is only one pair of integers (x, y) such that $x^2 + y^2 + 1 = xy + x + y$.
18. A painting fits snugly into a 1-inch frame of dimensions 5 inches by 11 inches. The painting covers more area than the frame.
19. The number one is the only integer to have a reciprocal that is also an integer.
20. Pierre de Fermat is most famous for proving his eponymous Last Theorem.
21. Quantum probability, tropical geometry, and universal algebra are all modern areas of mathematics.

22. Isaac Newton invented calculus, gravity, and Venn diagrams.
23. This room has less than 200 seats.
24. Suppose Victor writes problems at a rate of 10 problems every five days while John writes problems at a rate of 1 problem per day. If Victor and John collaborate, they will write a net total of 100 problems per month (assume a month consists of exactly 30 days). Victor and John will write more problems if they work together than if they work individually.
25. Most of the answers on the true and false section of this test are true.

2 Arithmetic

26. Evaluate $20 + 13$
27. Evaluate $13579 + 24680$
28. Evaluate $123 - 678$
29. What is $309 - 211$?
30. What is $1 + 2 + 3 + 4 + \ldots + 10$?
31. Evaluate 13×17
32. What is 35×34?
33. $2 * (3 + (3 + 2) * 5) + 1$
34. What is the sum of the distinct prime factors of 165?
35. What is $1001/7$?
36. What is the largest integer less than $32891/325$?
37. What is the remainder when 2013 is divided by 11?
38. Evaluate $\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\ldots\left(1 - \frac{1}{10}\right)$
39. Evaluate $\frac{1}{2} + \frac{1}{36} + \frac{1}{9}$
40. What is the minimum possible product of three different numbers in the set $\{-8, -6, -4, 0, 3, 5, 7\}$
41. What is $5^3 - 2^2 + 4^4$?
42. Let $P(x) = (x(x(x - 2) + 2) - 2) + 3$. What is $P(2)$?
43. What is $\binom{9}{3}$?
44. Evaluate $2^{100} \pmod{10}$
45. What is $\frac{1}{8} + \frac{2}{8^2} + \frac{3}{8^3} + \frac{4}{8^4} + \ldots$?

3 Counting/Probability

46. An apple tree grows an average of 2 apples per year. What is the average number of apples that will grow per year if I own 23 trees?

47. If you flip a two-sided coin three times, how many ways are there to get two heads and one tail?

48. At a conference with 5 people, each person shakes hands with every other person. How many total handshakes were there?

49. A room has 10 doors. How many ways can a person enter through one door and exit out a different door?

50. I choose an integer between 0 and 4 inclusive, and you choose an integer between 5 and 16 inclusive. What is the number of distinct sums we can come up with?

51. If you flip 2013 coins, what is the probability that more heads are showing than tails?

52. If you roll two six-sided dice, what is the probability that the sum of the two values rolled is 9?

53. If you roll two six-sided dice, what is the probability that the sum of the two values rolled is 9 given that one of the dice has a 2 on top?

54. How many diagonals are there in a octagon?

55. Two dice are thrown. What is the probability that the product of the two numbers is a multiple of 5?

56. What is the number of sets X such that $\{2, 4\}$ is a subset of X, and X is a subset of $\{1, 2, 3, 4, 5\}$?

57. There are four towns A, B, C, D in Wonderland. In Wonderland, all roads are one-way. Suppose there are 6 different roads from A to B, 3 from A to C, 4 from B to C and 2 from C to D. How many ways are there to get from A to C.

58. Using only pennies, nickels, dimes, and quarters, what is the smallest number of coins Henry would need so he could pay any amount of money less than a dollar?

59. How many ways are there to place 3 indistinguishable pegs on a 5x5 chessboard so that no two pegs lie in the same row or column?

60. A box contains two quarters. One is a double-headed coin, and the other is a fair coin, heads on one side, and tails on the other. You draw one coin from the box randomly, then look at one of the sides chosen at random. Given that the side you saw was a heads, what is the probability you have the fair coin?

4 Geometry

61. Henry and Lewin are standing on a point in 3D space. Henry flies away to a distance of 12 from the original point, and Lewin flies away to a distance of 5 from the original point. Let M be the maximum possible distance between Henry and Lewin, and let m be the minimum possible distance between Henry and Lewin. What is $M-m$?

62. A square has area 100. What is its perimeter?

63. The angles of a triangle sum to 180. What do the angles in a hexagon sum up to?

64. A right triangle has leg lengths 5 and 12. What is the length of the hypotenuse?

65. A right triangle has a hypotenuse of length 25 and a leg of length 24. What is the area of this triangle?

66. The area of an equilateral triangle is $2500\sqrt{3}$. What is the length of one side?

67. Say Henry is shaped like a square. If his friend, Lewin, is a circle with the same area as Henry, and Henry lies on his side, who is taller?

68. If the volume of a sphere is 36π, what is the surface area of the cube inscribed inside the sphere?

69. Points A, B, C, D are midpoints of sides of a larger square. If the larger square has area 60, what is the area of $ABCD$?

70. A regular tetrahedron has side length 1. What is its surface area?

71. In a regular hexagon, the distance from a vertex to the center is 6. What is the perimeter?

72. In a right triangle ABC with right angle at B, and altitude BD, we have $BC = 15$ and $CD = 9$. What is AB?

73. A sphere with radius 1 is inscribed within a cube with side length 2. What is the volume that is inside the cube but outside the sphere?

74. Suppose we have a triangle ABC and points D on AB and E on AC such that DE and BC are parallel. Suppose $DE = \frac{1}{3}BC$. What is the ratio of the length AD to DB?

75. Chords AB and CD of a given circle are perpendicular to each other and intersect at a right angle E. Given that $BE = 16, DE = 4$, and $AD = 5$, find CE.

5 Algebra

76. If $x = 2$ and $y = 3$, what is $2x + 3y$?

77. Given $2x + 12 = 32$, find x

78. If $a = -2$, what is the largest value of a number in the set $\{-3a, 4a, \frac{24}{a}, a^2, 1\}$?

79. Find the integer root of

$$P(x) = x(x(x-2)+2) - 4.$$

80. All the even numbers between 0 and 100 except for the ones ending in zero are multiplied together. What is the last digit in this product?

81. Let $a \otimes b = \frac{a+b}{a-b}$. What is $(6 \otimes 4) \otimes 3$?

82. Find the roots of the cubic polynomial $x^3 - 6x^2 + 11x - 6$.

83. Let $343^{42n} = 5$. What is the value of 49^{126n}?

84. Suppose Sally's pens all weigh the same, and her pencils all weigh the same. If it takes 25 pens to balance with 53 pencils, then at least how many pencils are needed to *outweigh* 100 pens?

85. What is the ratio of the number of questions in this section to the number of questions not in this section? Put your answer in a reduced fraction.

86. Let $f(x) = 1/x$ for $x > 0$. Find $f(f(3) + f(5))$

87. What is the smallest integer whose distinct prime factors add to 14?

88. Let a, b, c be nonzero real numbers such that $a + \frac{1}{b} = 5$, $b + \frac{1}{c} = 12$, and $c + \frac{1}{a} = 13$. Find $abc + \frac{1}{abc}$.

89. Let r_1, r_2, r_3 be the roots of $f(x) = x^3 - 6x^2 - 5x + 22$. Find $\frac{1}{r_1 r_2} + \frac{1}{r_2 r_3} + \frac{1}{r_3 r_1}$

90. Let x be the answer to this question. What is the sum of possible values of the average of the numbers $\{-16x, 3x^2, 10x\}$?

6 Miscellaneous

91. A frog is on the bottom of a 30 meter well. Each day, he has enough energy to jump up 10 meters. Each night, he slips 5 meters downward. How many days will it take for the frog to jump out?

92. How many numbers between 1 and 100 inclusive have an odd number of factors?

93. I have a bag filled with blue, red, or green coins. All but 4 of them are blue. All but 4 of them are green. All but 4 of them are red. How many coins do I have?

94. What is the minimum number of integers we need to draw between 1 and 100 inclusive such that we are guaranteed that the difference of two numbers is divisible by 3?

95. What is the maximum number of points we can put into a 1×1 square such that any pair of points is at least $\frac{1}{2}$ distance away from each other?

96. There are three boxes, one labeled APPLES, one labeled ORANGES, and one labeled APPLES AND ORANGES. Suppose all the labels on the boxes are wrong. You can pick only one item from one box to determine a way to fix the labels. What box do you choose from?

97. The answer to this question is twice the answer of the question below.

98. The answer to this question is one less than the answer of the above question.

99. How many times has the number 7 appeared on this speed round?

100. Let $x < 100$ be the answer to this question. What is the answer to the xth question of this test?