1. An *exterior angle* is the supplementary angle to an interior angle in a polygon. What is the sum of the exterior angles of a triangle and dodecagon (12-gon), in degrees?

   Answer: 720

   Solution: For any polygon, the sum of the exterior angles is $360°$. The sum of the exterior angles of a triangle and dodecagon are each $360°$, and $360° \cdot 2 = \boxed{720}^{\circ}$.

2. Let $\eta \in [0, 1]$ be a relative measure of material absorbence. η values for materials combined together are additive. η for a napkin is 10 times that of a sheet of paper, and a cardboard roll has $\eta = 0.75$. Justin can create a makeshift cup with $\eta = 1$ using 50 napkins and nothing else. How many sheets of paper would he need to add to a cardboard roll to create a makeshift cup with $\eta = 1$?

   Answer: 125

   Solution: Since 50 napkins are necessary to create a makeshift cup with $\eta = 1$, each napkin has $\eta = \frac{1}{50} = 0.02$. Then each paper would have $\eta = \frac{1}{10}$ of that of each napkin, or $\eta = 0.002$. To create a makeshift cup using a cardboard roll and sheets of paper, Justin would need the paper to have a cumulative η value of $1 - 0.75 = 0.25$. Therefore, he would need $\frac{0.25}{0.002} = \boxed{125}$ sheets of paper to create the makeshift cup.

3. $\triangle ABC$ has $AB = 5$, $BC = 12$, and $AC = 13$. A circle is inscribed in $\triangle ABC$, and $\overline{MN}$ tangent to the circle is drawn such that M is on $\overline{AC}$, N is on $\overline{BC}$, and $\overline{MN} \parallel \overline{AB}$. The area of $\triangle MNC$ is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

   Answer: 43

   Solution: Using the formula $A = rs$, where r denotes inradius, s denotes semiperimeter, and A denotes area, $r = \frac{A}{s} = \frac{30}{15} = 2$. Then, because the diameter of the incircle of $\triangle CAB$ incircle is 4, $BN = 4$ so $CN = 8$. Because $\triangle CMN \sim \triangle CAB$ by AA similarity, the ratio of their sides is $\frac{2}{3}$, so the area of $\triangle MNC$ is $\frac{4}{9}$ that of $\triangle ABC$. Then the area of $\triangle MNC$ is $\frac{4}{9}\left(\frac{1}{2} \cdot 5 \cdot 12\right) = \frac{40}{3}$, and our answer is $\boxed{43}$.

4. In an 6×6 grid of lattice points, how many ways are there to choose 4 points that are vertices of a nondegenerate quadrilateral with at least one pair of opposite sides parallel to the sides of the grid?

   Answer: 6525

   Solution: First, we count the quadrilaterals with horizontal bases. There are $\binom{6}{2} = 15$ ways to choose the y-coordinates of the bases. Then, there are $\binom{6}{2}^2 = 225$ ways to choose the coordinates of the quadrilateral's vertices. Similarly, there are $15 \cdot 225$ ways to create quadrilaterals with vertical bases. However, we have counted the rectangles twice, so we must subtract $\binom{6}{2}^2 = 225$ rectangles. So there are a total of $15 \cdot 225 + 15 \cdot 225 - 225 = 29 \cdot 225 = \boxed{6525}$ nondegenerate quadrilaterals that satisfy the given constraint.

5. The polynomial $f(x) = x^3 + rx^2 + sx + t$ has r, s, and t as its roots (with multiplicity), where $f(1)$ is rational and $t \neq 0$. Compute $|f(0)|$.

   Answer: 1

   Solution: First, we have by Vieta's formulae that $rst = -t$. Since $t \neq 0$, $rs = -1$, so we write

$$s = -\frac{1}{r}.$$

Now we also observe (from Vieta's formulae) that $r+s+t=-r$, so $t=-2r-s=-2r+\frac{1}{r}$. Now we can write

$$\begin{aligned} f(x) &= (x-r)(x-s)(x-t) \\ &= (x-r)\left(x+\frac{1}{r}\right)\left(x+2r-\frac{1}{r}\right) \\ &= x^3+rx^2-\left(2r^2-2+\frac{1}{r^2}\right)x-2r+\frac{1}{r} \\ &= x^3+rx^2+sx+t \\ &= x^3+rx^2-\frac{1}{r}x-2r+\frac{1}{r}. \end{aligned}$$

Equating the coefficients of x in the third and fifth lines above yields $\frac{1}{r}=2r^2-2+\frac{1}{r^2}$, so $2r^4-2r^2-r+1=0$. We are given that

$$f(1)=r+s+t+1=-r+1$$

is rational, so r must be rational. By the rational root theorem, the only possible values for r are ± 1 and $\pm\frac{1}{2}$. A simple check reveals that $r=1$ is the only possibility, whence we find

$$f(x)=x^3+x^2-x-1,$$

so $|f(0)|=\boxed{1}$.