1. For all a and b, let $a \clubsuit b = 3a + 2b + 1$. Compute c such that $(2c) \clubsuit (5 \clubsuit (c+3)) = 60$.

   Answer: $\frac{3}{2}$

   Solution: Since

$$\begin{aligned}(2c) \clubsuit (5 \clubsuit (c+3)) &= (2c) \clubsuit (3(5) + 2(c+3) + 1) \\ &= (2c) \clubsuit (2c + 22) \\ &= 3(2c) + 2(2c+22) + 1 \\ &= 10c + 45,\end{aligned}$$

   we require $10c + 45 = 60$, so $c = \boxed{\frac{3}{2}}$.

2. Suppose that $(i-1)^{11}$ is a root of the quadratic $x^2 + Ax + B$ for integers A and B, where $i = \sqrt{-1}$. Compute the value of $A + B$.

   Answer: 1984

   Solution: Recall $\text{cis}(\theta) = \cos\theta + i\sin\theta$. By Euler's formula,

$$(i-1)^{11} = (\sqrt{2} \cdot \text{cis}(3\pi/4))^{11} = 2^{11/2} \cdot \text{cis}(\pi/4) = 32 + 32i.$$

   Because A and B are real, it follows that $x^2 + Ax + B$ has roots $32 \pm 32i$ since the complex conjugate must also be a root. By Vieta's formulas, $A = -64$ and $B = 2048$, so $A + B = \boxed{1984}$.

3. Tej writes $2, 3, \ldots, 101$ on a chalkboard. Every minute he erases two numbers from the board, x and y, and writes $xy/(x+y-1)$. If Tej does this for 99 minutes until only one number remains, what is its maximum possible value?

   Answer: $\frac{101}{100}$

   Solution: Let the numbers at any given time be represented by $\{a_i\}$; for instance, initially, $a_1 = 2, a_2 = 3, \ldots, a_{100} = 101$. For each i, let $b_i = (1 - a_i^{-1})^{-1}$. This yields $a_i = (1 - b_i^{-1})^{-1}$ and $a_j = (1 - b_j^{-1})^{-1}$ which imply

$$\begin{aligned}\frac{a_i a_j}{a_i + a_j - 1} &= \left(a_i^{-1} + a_j^{-1} - (a_i a_j)^{-1}\right)^{-1} \\ &= \left((1 - b_i^{-1}) + (1 - b_j^{-1}) - (1 - b_i^{-1})(1 - b_j^{-1})\right)^{-1} \\ &= \left(1 - (b_i b_j)^{-1}\right)^{-1}\end{aligned}$$

   In particular, $\prod b_i$ is invariant and Tej's final number is

$$\left(1 - \prod_i b_i^{-1}\right)^{-1} = \left(1 - \prod_i (1 - a_i^{-1})\right)^{-1} = \left(1 - \prod_{i=1}^{100} \frac{i}{i+1}\right)^{-1} = \boxed{\frac{101}{100}}.$$