Time limit: 60 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. Compute the three-digit number that satisfies the following properties:
   - The hundreds digit and ones digit are the same, but the tens digit is different.
   - The number is divisible by 9.
   - When the number is divided by 5, the remainder is 1.

2. Three people, Pranav, Sumith, and Jacklyn, are attending a fair. Every time a person enters or exits, the groundskeeper writes their name down in chronological order. If each person enters and exits the fairgrounds exactly once, in how many ways can the groundskeeper write down their names?

3. Find the number of positive integers n less than 10000 such that there are more 4's in the digits of $n + 1$ than in the digits of n.

4. Given positive integers $a \geq 2$ and k, let $m_a(k)$ denote the remainder when k is divided by a. Compute the number of positive integers, n, less than 500 such that $m_2(m_5(m_{11}(n))) = 1$.

5. Kait rolls a fair 6-sided die until she rolls a 6. If she rolls a 6 on the Nth roll, she then rolls the die N more times. What is the probability that she rolls a 6 during these next N times?

6. Let N be the number of positive integers x less than $210 \cdot 2023$ such that

$$\operatorname{lcm}(\gcd(x, 1734), \gcd(x + 17, x + 1732))$$

   divides 2023. Compute the sum of the prime factors of N with multiplicity. (For example, if $S = 75 = 3^1 \cdot 5^2$, then the answer is $1 \cdot 3 + 2 \cdot 5 = 13$).

7. Maria and Skyler have a square-shaped cookie with a side length of 1 inch. They split the cookie by choosing two points on distinct sides of the cookie uniformly at random and cutting across the line segment formed by connecting the two points. If Maria always gets the larger piece, what is the expected amount of extra cookie in Maria's piece compared to Skyler's, in square inches?

8. Define a family of functions $S_k(n)$ for positive integers n and k by the following two rules:

$$S_0(n) = 1,$$

$$S_k(n) = \sum_{d|n} dS_{k-1}(d).$$

   Compute the remainder when $S_{30}(30)$ is divided by 1001.

9. Shiori places seven books, numbered from 1 to 7, on a bookshelf in some order. Later, she discovers that she can place two dividers between the books, separating the books into left, middle, and right sections, such that:
   - The left section is numbered in increasing order from left to right, or has at most one book.

- The middle section is numbered in decreasing order from left to right, or has at most one book.
- The right section is numbered in increasing order from left to right, or has at most one book.

In how many possible orderings could Shiori have placed the books? For example, $(2,3,5,4,1,6,7)$ and $(2,3,4,1,5,6,7)$ are possible orderings with the partitions $2,3,5|4,1|6,7$ and $2,3,4|1|5,6,7$, but $(5,6,2,4,1,3,7)$ is not.

10. Let α denote the positive real root of the polynomial x^2-3x-2. Compute the remainder when $\lfloor \alpha^{1000} \rfloor$ is divided by the prime number 997. Here, $\lfloor r \rfloor$ denotes the greatest integer less than r.